

February 9, 2008

## “Chaos” in Nuclear High Spin Spectroscopy

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### Abstract

The Projected Shell Model with zero-, two- and four-quasiparticle configurations is used to investigate the level statistics, i.e. chaoticity, of high spin spectroscopy. The model can describe many high spin phenomena and with the present configuration space has sufficient number of levels for statistical analysis. It is found that the degree of chaoticity has a sensitive dependence on the classification of levels in question, and that it steadily increases with excitation energy and angular momentum.

Level-spacing statistics, *a la* Random Matrix Theory [1] is an important tool to study nuclear spectroscopy. In the past decade, it has also gradually emerged as one of the workable definitions of order and chaos for a quantum system, such as nucleus [2]. Although there is no commonly accepted definition of “quantum chaos” [3], there is no doubt that level-spacing statistics does produce two limiting distributions: Poisson and GOE (Gaussian Orthogonal Ensemble), which, for a large class of models, correspond to ordered and chaotic classical motions, respectively. It would be interesting to know “in which windows of the parameter space (energy, angular momentum, parity *etc.*) will the distribution of the nuclear levels be GOE-like”. If this question can be answered, and if one could firmly establish that GOE distribution is indeed a manifestation of “quantum chaos”, then one can further investigate questions such as “how does a nuclear system become chaotic”. This Letter will address the former question.

About a decade ago, Hag, Pandey and Bohigas [4] began to search for such windows in the nuclear system. However, they were immediately faced with a difficulty which persisted to this date: The number of available data is well below what is required for a statistical analysis [4,5]. To remedy this, they performed the statistical analysis on groups of states collected from different nuclei, the nuclear data ensemble. However, Abul-Magd and Weidenmüller realized early on that the outcome of the statistical analysis can be dependent on the classification and selection of the data. They showed that one can obtain different distributions for different groups of the same set of data [6]. Still, this important idea is somewhat compromised since they, faced with the aforementioned shortage of data, were unable to separate them according to parity and angular momentum.

With the empirical statistical analysis stalled, to answer the above mentioned question will, within the foreseeable future, rely on theoretical models. Of course, this line of research will be intimately linked to the question of how well such models can reproduce the known data and can construct sufficient number of levels. Recently, Alhassid and Vretenar [7] and Wu, Feng and Vallières [8] made significant steps toward this direction by using, respectively, the Interacting Boson Model (IBM) and the Fermion Dynamical Symmetry Model (FDSM)

[9]. While the latter concentrated only on the question of dynamical symmetry breaking in low energy spectroscopy, the former also studied the statistical behavior of the spectroscopy at high spins. It deserved mentioning that none of these approaches has any realistic nuclei in mind and that the suitability of the IBM for high spin states is still very much a topic of current investigation.

It is therefore highly desirable to have a model which can reproduce well the spectroscopy of high spin states to carry out the level-spacing statistics. Straightforward implementation of the spherical shell model is naturally out of question for the heavy systems. The ambitious approach (MONSTER) developed by the Tuebingen group [10] could in principle be used, but since for practical reasons the configurations are restricted to 2-qp states, it may not be sufficient to study the statistical behavior of levels in higher excitation energies. The FDSM [9] can also in principle allow such studies be carried out, and its implementation is currently underway. Quite recently, a model called the Projected Shell Model (PSM), proposed in the late 1970s [11] by the Munich group, has undergone extensive development [12,13]. A code for this model was developed and successfully used to study a range of high spin phenomena in the rare earths. It is thus a practical shell model approach to describe the deformed heavy systems.

Since the PSM has been discussed in several publications, and is the subject matter of a forthcoming review article [14], we shall only touch upon the relevant features here. Roughly speaking, unlike the conventional shell model, the PSM begins with the deformed (Nilsson-type [15]) single particle basis. Its advantage over the conventional shell model is that the important nuclear correlations are easily taken into account and the configuration space is manageable, thus making the shell model treatment for heavy systems possible. Also, it deserves to be emphasized that the results obtained from the PSM can be interpreted in simple physical terms. Such a shell model basis violates the rotational symmetry, but it can be restored by the standard angular momentum projection technique. Pairing correlation is included by a successive BCS calculation for the Nilsson states. Thus, the shell model truncation is carried out within the quasiparticle states with the vacuum  $|\phi\rangle$ .

The angular momentum projected wave function for the PSM is given by  $|IM\rangle = \sum_{\kappa} f_{\kappa} \hat{P}_{MK_{\kappa}}^I |\varphi_{\kappa}\rangle$ , where  $\kappa$  labels the basis states. Here we shall assume axial symmetry in the intrinsic states  $|\varphi_{\kappa}\rangle$ . Thus  $K$  is a good quantum number. Using the Tamm-Dancoff-Approximation [16], the basis states  $|\varphi_{\kappa}\rangle$  for a doubly even system are spanned by

$$\left\{ |\phi\rangle, \alpha_i^{\dagger}\alpha_j^{\dagger}|\phi\rangle, \alpha_i^{\dagger}\alpha_j^{\dagger}\alpha_k^{\dagger}\alpha_l^{\dagger}|\phi\rangle, \dots \right\}, \quad (1)$$

where  $\{\alpha, \alpha^{\dagger}\}$  are the quasiparticle annihilation and creation operators for the vacuum  $|\phi\rangle$ . To restore the rotational symmetry, these intrinsic state  $|\varphi_{\kappa}\rangle$  will be acted on by the projection operator  $\hat{P}_{MK}^I$  [16]. This will generate states of good angular momentum. For example,  $\hat{P}_{MK}^I|\phi\rangle$  will describe the ground-state (g-) band. Thus, the model has the inherent flexibility that by including higher order multi-quasiparticle states in eq. (1), one can reach levels of higher excitation energy and angular momentum. In fact, if one were to construct all possible configurations in eq. (1), then one will reach the exact shell model space. However, for reproducing the yrast properties, a basis with 60 low-lying configurations is sufficient [12].

In this work, the single particle space is sufficiently large and consists of three major shells: i.e.  $N = 4, 5$  and  $6$  ( $N = 3, 4$  and  $5$ ) for neutrons (protons). The multi-quasiparticle basis states of eq. (1) are constructed by both the normal and the abnormal parity orbitals. We should also point out that the 4-qp states are built from a neutron and a proton pairs. Such states will in general lie lower in energy than the like-particle (protons and neutrons) 4-qp states. The latter states are not included in the present work. Also, the size of the basis states of eq. (1) is determined by using the (unprojected quasiparticle) energy cut-off of 5 MeV for both 2- and 4-qp states. Within this energy window, there will be about 200 2-qp states from the  $N = 5$  and  $6$  neutron and  $N = 4$  and  $5$  proton major shells. Likewise, about 500 4-qp states based on those 2-qp states are constructed. So the dimension of the basis is about 700. It was pointed out by Åberg [17] that the 6-qp states will begin to play a role at the (super-deformed high rotating) particle-hole excitation energy in the vicinity

of 4 MeV. The possible effect of the missing 6-qp and the like-particle 4-qp configurations will be discussed later. Also, the following hamiltonian is used [11]:

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_{\mu} \hat{Q}_{\mu}^{\dagger} \hat{Q}_{\mu} - G_M \hat{P}^{\dagger} \hat{P} - G_Q \sum_{\mu} \hat{P}_{\mu}^{\dagger} \hat{P}_{\mu}. \quad (2)$$

This is essentially the well-known Pairing plus Quadrupole Hamiltonian [18] which is known to describe not only the nuclear ground-state properties but the region at finite temperature as well [19]. In addition, there is also a quadrupole pairing term whose importance was recently once again demonstrated [20]. Yet, despite the simplicity, this hamiltonian has worked surprisingly well in predicting various high spin phenomena [12,13,20,21]. The interaction strengths are determined as follows: the quadrupole interaction strength  $\chi$  is adjusted so that the known quadrupole deformation  $\epsilon_2$  is obtained from the Hartree-Fock-Bogoliubov self-consistent procedure [22]. The monopole pairing strength  $G_M$  is adjusted to the known energy gap  $G_M = \left[20.12 \mp 13.13 \frac{N-Z}{A}\right] \cdot A^{-1}$ , where the minus (plus) sign is for neutrons (protons). The quadrupole pairing strength  $G_Q$  is assumed to be proportional to  $G_M$  and the proportional constant is fixed at 0.16 in this work. Diagonalization of the Hamiltonian of eq. (2) in the truncated projected basis of eq. (1) is equivalent to mixing states with different  $K$  quantum numbers. For a given angular momentum  $I$  and parity  $\pi$ , one will finally obtain a set of energy levels  $\{E_i\}$ . The highest state obtained after diagonalization for each  $I$  can reach an excitation region of 7 MeV above the yrast line. The total number of obtained levels is typically 460 (650) for  $I = 4\hbar$  ( $I = 10\hbar$ ).

To illustrate the physics, we have chosen a typical backbender  $^{164}\text{Er}$ . This is a well deformed nucleus with a purely rotational ground band. It was previously shown [12] that with the same model, one can well describe the high spin phenomena in the yrast region of this and many other deformed nuclei. Hence there is no loss of generality to choose this nucleus as an example. We shall first focus our discussion on the level statistics as a function of excitation energy for a given angular momentum ( $I = 10\hbar$ ) and positive parity. In the top part of Fig.1, the statistics for the entire set of levels for this spin and parity are given. From both the nearest level-spacing distribution  $P(X)$  and the  $\Delta_3(L)$  statistics we see that

the distribution lies somewhere between the Poisson and GOE limits. As we divide them into groups according to excitation energies, clear pictures of Poisson and GOE distributions emerge. In the middle part of Fig.1, the group of levels are those up to 2 MeV excitation energy above the yrast line. Here both  $P(X)$  and  $\Delta_3(L)$  are manifestly Poisson-like. When only states of higher excitation energies are included ( $4 \rightarrow 6$  MeV), GOE statistics appears (see bottom part of Fig.1).

It should be emphasized that the mixing of  $K$ -states by the two-body residual interactions play a key role in understanding the above results. The projected quasiparticle basis forms a spin/parity group, with each state having a definite  $K$ . Hence diagonalization will mix the  $K$ -states and the degree of the  $K$ -mixing can differ for the different excitation and spin regions. Clearly, there will be strong mixing between bands lying close in energy and with the same symmetry, which is the case for the region of high excitation energies. On the other hand, at low excitations,  $K$  is approximately a good quantum number because bands are well separated. Hence our analyses suggest that the level-spacing statistics could be an indication of the amount of  $K$ -mixing for a certain group of levels in well deformed nuclei, namely, the Poisson statistics will signal a weaker and the GOE a stronger mixing of the  $K$ -states. This can also explain why the top part of Fig.1 is an intermediate situation since the group of states in question includes both stronger and weaker  $K$ -mixing ones.

To enhance the above idea, we shall show in Fig.2 four data groups. These are groups of the calculated  $I^\pi = 10^+$  levels of  $^{164}\text{Er}$ , each with a different energy limit at the top (i.e.  $0 \rightarrow 3$ ,  $0 \rightarrow 4$ ,  $0 \rightarrow 5$  and  $0 \rightarrow 6$  MeV). Together with the middle part of Fig.1, we see that there is a smooth transition from Poisson to GOE, thus allowing us to speculate that with increase in energy, the strong mixing component will eventually dominate the statistics, resulting in a pure GOE picture.

We now turn our attention to the dependence of the statistics on nuclear rotation, or explicitly, angular momentum. It is well-known from the phenomenological argument that the Coriolis force tends to mix the symmetry of a rotating system. With larger angular momentum, the force will of course increase as well. In the microscopic model, it can be

shown that at higher spins, level density is high already near the yrast region [12]. Therefore, it is easy to conjecture that level-spacing distribution will steadily approach the GOE limit for sufficiently rapid rotation. Our calculations here do support this conjecture (see the plots of the left row in Fig.3).

In a recent publication [7], Alhassid and Vretenar used the IBM with one broken pair to demonstrate that the GOE distribution is best manifested in the vicinity of the crossing between the ground and the lowest 2-qp bands. Beyond it, the Poisson statistics appears to be restored. This is in direct contradiction to the results we obtained here. To resolve this discrepancy, we extracted from eq. (1) only the vacuum and the 2-qp states built by the  $i_{13/2}$  neutrons. This is thus a caricature of the model used in ref. [7]. The size of this basis is 70. It should be mentioned that with such a basis, the yrast band of  $^{164}\text{Er}$  can be adequately described and the backbending at spin  $16\hbar$  is reproduced exactly [12]. The results for this basis are shown on the right row of plots in Fig.3. As expected, the results are consistent with ref. [7], i.e. the distribution is GOE-like at the band crossing region and approaches Poisson for higher spins. These results suggest that the conclusion of Alhassid and Vretenar could be due, at least in part, to the lack of 4-qp components in their calculation. It also implies that our results (the left row of Fig.3) could be more GOE-like when the like-particle 4- as well as 6-qp configurations are included.

In the present study, the angular momentum is treated fully quantum mechanically. However, the particle number is conserved only on the average. Therefore, our conclusions contain the assumption that statistics of the neighboring nuclei, especially in the well deformed region, should not differ much in character. Although there is no evidence against this assumption, it should nevertheless be checked in later studies.

To conclude, we have extended the application of the Projected Shell Model, which has been successfully used to describe the high spin spectroscopy of the yrast region, to the higher excitation regions. The level-spacing statistics was carried out by using up to 700 0-, 2- and 4-qp levels, reaching as high as 7 MeV above the yrast line. We showed that the resulting statistics has a strong dependence on how the levels are classified. Steady transition

from Poisson-like to GOE-like was found as one moves from low to high excitation energy and/or from low to high spin. It appears that we have thus answered the question, stated in the beginning of this Letter, of “in which windows of the parameter space the distribution of the nuclear levels will become GOE-like”. Clearly, we see that the presence of the GOE spectroscopy seems to satisfy much of our intuitive understanding of chaos. However, for a deeper understanding, one must answer the next question of “how a nuclear system becomes chaotic”. Yet, the final link between GOE and chaos, a central question of the mysterious field of “quantum chaos” remains to be made.

### **ACKNOWLEDGMENTS**

Useful discussions with P. Ring, T. von Egidy and S. Åberg are acknowledged. Yang Sun is most grateful to the College of Arts and Science of Drexel University for the provision of a research fellowship. This work is partially supported by the United States National Science Foundation.



## REFERENCES

- [1] M.L. Metha, *Random Matrices and the Statistical Theory of Energy Levels* (Academic Press, New York, 1967)
- [2] O. Bohigas et al, Phys. Rev. Lett. **52**, 1 (1984); O. Bohigas and H.A. Weidenmüller, Ann. Rev. Nucl. Part. Sci. **38**, 421 (1988); Y. Alhassid et al, Phys. Rev. Lett. **65**, 2971 (1990); S. Åberg, Prog. Part. Nucl. Phys. **28**, 11 (1992); B.R. Mottelson, Nucl. Phys. **A557**, 717c (1993)
- [3] N.L. Balazs and A. Voros, Phys. Rep. **143**, 109 (1986); T. Cheon and T.D. Cohen, Phys. Rev. Lett. **62**, 2769 (1989); H. Wu et al, Phys. Rev. **A42**, 1027 (1990); V.G. Zelevinsky, Nucl. Phys. **570**, 411c (1994)
- [4] R.V. Hag, A. Pandey and O. Bohigas, Phys. Rev. Lett. **48**, 1086 (1982)
- [5] T. von Egidy et al, Nucl. Phys. **A454**, 109 (1986); *ibid*, **A481**, 189 (1988); G.E. Mitchell et al, Phys. Rev. Lett. **61**, 1473 (1988); S. Raman et al, Phys. Rev. **C43**, 521 (1991); J.D. Garrett and J.R. German, Phys. Rev. Lett., submitted
- [6] Y.A. Abul-Magd and H.A. Weidenmüller, Phys. Lett. **162B**, 223 (1985)
- [7] Y. Alhassid and D. Vretenar, Phys. Rev. **C46**, 1334 (1992)
- [8] H. Wu, D.H. Feng and M. Vallières, J. Phys. **G16**, L149 (1990)
- [9] C.-L. Wu, D.H. Feng and M. Guidry, Adv. Nucl. Phys. **21**, 227 (1994)
- [10] K.W. Schmid and F. Gruemmer, Rep. Prog. Phys. **50**, 731 (1987)
- [11] K. Hara and S. Iwasaki, Nucl Phys. **A348**, 200 (1980)
- [12] K. Hara and Y. Sun, Nucl Phys. **A529**, 445 (1991)
- [13] Y. Sun and J.L. Egido, Phys. Rev. C, in press
- [14] K. Hara and Y. Sun, to be published
- [15] C.G. Andersson et al, Nucl. Phys. **A309**, 41 (1978)
- [16] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer-Verlag, Berlin, 1980)
- [17] S. Åberg, Nucl. Phys. **A477**, 18 (1988)
- [18] M. Baranger and K. Kumar, Nucl. Phys. **A110**, 490 (1968)
- [19] J.L. Egido and P. Ring, J. Phys. **G19**, 1 (1993)
- [20] Y. Sun, S. Wen and D.H. Feng, Phys. Rev. Lett. **72**, 3483 (1994)
- [21] Y. Sun, D.H. Feng and S. Wen, Phys. Rev. **C**, submitted
- [22] I.L. Lamm, Nucl. Phys. **A125**, 504 (1969)

## FIGURES

FIG. 1. Statistics for the unfolded energy levels of  $I^\pi = 10^+$ . The plots on the left row show the nearest-neighbor level spacing distribution  $P(X)$  and those on the right the  $\Delta_3$  statistics. For both rows, the dotted (dashed) curves correspond to Poisson (GOE) distribution. 1) Top: the entire set of levels (650). 2) Middle: Levels from the yrast state up to 2 MeV. 3) Bottom: Levels from 4 to 6 MeV.

FIG. 2. Statistics for the unfolded energy levels of  $I^\pi = 10^+$  for different level classifications. The descriptions of the curves are the same as in Fig.1.

FIG. 3. The  $\Delta_3$  statistics as a function of angular momentum. The dotted (dashed) curves correspond to Poisson (GOE) distribution. 1) Left row: statistics for the entire set of levels obtained from the whole basis at each spin. 2) Right row: statistics for the restricted basis. For details, see the text.

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